# Employment, Hours of Work and the Optimal Taxation of Low Income Families

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**Abstract**

The optimal design of low income support is examined using a structural labour supply model. The approach incorporates unobserved heterogeneity, fixed costs of work, childcare costs and the detailed non-convexities of the tax and transfer system. The analysis considers purely Pareto improving reforms and also optimal design under social welfare functions with different degrees of inequality aversion. We explore the gains from tagging and also examine the case for the use of hours-contingent payments. Using the tax schedule for lone parents in the UK as our policy environment, the results point to a reformed non-linear tax schedule with tax credits only optimal for low earners. The results also suggest a welfare improving role for tagging according to child age and for hours-contingent payments, although the case for the latter is mitigated when hours cannot be monitored or recorded accurately by the tax authorities.

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# 1 Introduction

This paper develops a structural approach to the optimal design of low income support. The analysis concerns the optimal choice of the tax rate schedule in a Mirrlees (1971) framework extended to allow for unobserved heterogeneity, fixed costs of work, childcare costs and the detailed non-convexities of the tax and transfer system. Within this framework we consider Pareto improving reforms. We also explore the implications for the optimal tax rate schedule under social welfare functions with different degrees of inequality aversion.

The tax treatment of lone parents in the UK is used as the empirical environment for our analysis.[1] As in North America, this group has been the subject of a number of tax and benefit reforms, see Blundell and Hoynes (2004). These reforms can provide useful variation for assessing the reliability of structural models. In particular, we use the 1999 Working Families' Tax Reform (WFTC) in the UK which considerably increased the generosity of in-work benefits/tax credits for lone parents, see Brewer (2001). The WFTC programme uses hours-contingent payments.[2] Eligibility requires parents to be working in a job with at least 16 hours of work per week.

Designing low income support is complicated. How should taxes and transfers depend on income when taking into account the labour supply responses for this group involving both intensive and extensive margin responses? Tagging has been suggested to improve the trade-off between equality and efficiency but how large are the potential gains? Hours are partially observable and used in practise for low-income support but how good is this "signal"? Optimal tax theory points to the relevant trade-offs but we need solid measurement of these trade-offs in order to move from theory to practical policy recommendations on how to reform actual tax schedules. The paper bridges the gap by setting up a structural model that is able to address all of these questions.

The microeconometric analysis here is based on an extension of the stochastic discrete choice labour supply approach (Hoynes, 1996; Keane and Moffitt, 1998;

[1]The Mirrlees Review provides a recent overview of UK earnings tax design (Brewer et al., 2010).

[2]Hours conditions are used in the tax credit systems in Ireland and New Zealand. They are also proposed in Keane (1995), although not within an optimal tax framework.

Blundell et al., 2000; van Soest et al., 2002). This approach allows us to distinguish between the intensive margin of hours of work and the extensive margin where the work decision is made. As the empirical literature on labour supply has demonstrated, labour supply elasticities for certain groups of working age individuals appear to be much larger at the extensive margin, see Blundell and Macurdy (1999), for example. As Saez (2002) and Laroque (2005) have shown, empirical results on the responsiveness of different types of individuals at different margins of labour supply have strong implications for the design of earnings taxation.

Consistent with the empirical literature on the labour supply of the low paid, our structural estimation results show important differences in the responsiveness of labour supply at different margins. We use our estimated model to identify inefficiencies in the actual tax and transfer system and characterise Pareto improving reforms. This analysis points to relatively minor improvements in the tax schedule for lone parents. When imposing a social welfare function with reasonable social welfare weights, we obtain a reformed non-linear tax schedule with lower tax rates over a large range of earnings for many families, and with tax credits only optimal for low earners.

We also find that labour supply responses vary according to the age of children. We use this variation to quantify the potential welfare gains from tagging according to child age. Our results suggest a welfare improving role for age-based tagging, with tax credits being found to be most important for low earning families with school age children. Our results also point to welfare gains from using hours-contingent payments. If the tax authorities are able to choose the lower limit on working hours that trigger eligibility for such families, we present an empirical case for using a full-time work rule rather than the main part-time rule currently in place for parents in the UK. However, the case is substantially mitigated when hours cannot be monitored or recorded accurately by the tax authorities.

The remainder of the paper proceeds as follows. In the next section we develop the analytical framework for optimal design within a stochastic structural labour supply model. In section 3 we outline the WFTC reform in the UK and its impact

on work incentives. Section 4 details the structural microeconometric model, while in section 5 we describe the data and model estimates. Section 6 uses these model estimates to explore what normative conclusions may be derived from a weak Pareto improvement criteria. In section 7 we then consider what additional results may be derived by imposing a specific social welfare function; we also demonstrate how these optimal tax schedules vary when we allow for tagging by age of children, and when hours of work are included in the tax base. Finally, section 8 concludes.

## 2 The Optimal Design Problem

In this section we develop the analytical framework that will be used to explore both Pareto improving reforms to the actual tax and transfer system, as well as tax reforms that are optimal under some social welfare function.[3] In both cases we will allow for two scenarios. In the first only earnings and employment are observable by the tax authority, in the second we also allow for partial observability of hours of work. Hours of work $h$ are chosen from the finite set $\mathcal{H} = \{h_0, \dots h_J\}$, with partial observability incorporated by allowing the tax authorities to additionally observe that hours belong to some closed interval $\mathbf{h} = [\underline{h}, \overline{h}] \in \mathcal{H}$ with $\underline{h} \leq h \leq \overline{h}$. For example, the tax authorities may be able to observe whether individuals are working at least $h_B$ hours per week, but conditional on this, not how many.[4]

Work decisions by individuals (single mothers) are determined by their preferences over consumption $c$ and labour hours $h$, as well as possible childcare requirements, fixed costs of work, and the tax and transfer system. Preferences are indexed by observable characteristics $X$, including the number and age of her children, and vectors of unobservable (to the econometrician) characteristics $\epsilon$ and $\varepsilon$. The vector $\varepsilon$ is independent of both $X$ and $\epsilon$ and corresponds to the additive hours (or state) specific

[3] An alternative model which incorporates constraints on the labour supply choices in an optimal design problem is developed in Aaberge and Colombino (2008).

[4] Depending on the size of the interval, this framework nests two important special cases; (i) when hours are perfectly observable $\underline{h} = \overline{h} = h$ for all $h \in \mathcal{H}$; (ii) only earnings information is observed $\mathbf{h} = \mathcal{H}_{++}$ for all $h > 0$. In general this is viewed as a problem of partial observability since actual hours $h$ are always contained in the interval $\mathbf{h}$. In our later analysis in section 7.4 we will explore the effect that both random hours measurement error, and possible hours misreporting have upon the optimal design problem.

errors in the utility function; we let $U(c,h;X,\epsilon,\varepsilon) = u(c,h;X,\epsilon) + \varepsilon_h$ represent the utility of a single mother who consumes $c$ and works $h$ hours. We will assume that she consumes her net income which comprises the product of hours of work $h$ and the gross hourly wage $w$ plus non-labour income and transfer payments, less taxes paid, childcare expenditure, and fixed costs of work. In what follows we let $F$ denote the cumulative distribution function of the state specific errors $\varepsilon$, and let $G$ denote the joint cumulative distribution function of $X$ and $\epsilon$. We assume that $\varepsilon$ is independent of both $\epsilon$ and $X$.

In our empirical analysis individual utilities $U(c,h;X,\epsilon,\varepsilon)$ will be described by a parametric utility function and a parametric distribution of unobserved heterogeneity $(\epsilon,\varepsilon)$. Similarly, a parametric form will be assumed for the process determining fixed costs of work and childcare expenditure. To maintain focus on the design problem, we delay this discussion regarding the econometric modelling until section 4. For now it suffices to write consumption $c$ at hours $h$ as $c(h;T,X,\epsilon)$[5] where $T(wh,\mathbf{h},X)$ represents the tax and transfer system. Non-labour income, such as child maintenance payments, enter the tax and transfer schedule $T$ through the set of demographics $X$, and for notational simplicity we abstract from the potential dependence of the tax and transfer system on childcare expenditure. Taking $T$ as given, each single mother is assumed to choose her hours of work $h^* \in \mathcal{H}$ to maximise her utility. That is:

$$h^* = \arg\max_{h\in\mathcal{H}} U(c(h;T,X,\epsilon),h;X,\epsilon,\varepsilon). \tag{1}$$

## 2.1 Pareto Improving Reforms

The first stage of our optimal design analysis explores the normative conclusions that may be derived from a Pareto improvement criterion. This exercise is closely related to Werning (2007), which characterized the set of Pareto efficient tax schedules within the Mirrlees (1971) model, and which proposed a simple test for the efficiency of a given tax schedule through the lens of that model.

To explore the efficiency of a given tax and transfer system $T_e$ we first calculate the

[5]Conditional on work hours $h$, consumption will not depend on $\varepsilon$ given our assumption that $\varepsilon$ enters the utility function additively and is independent of $(X,\epsilon)$.

(incentive compatibility) maximised value of utility under this system for all $(X, \epsilon, \varepsilon)$. With slight abuse of our above notation, we denote these maximised utility levels as $U(T_e, X, \epsilon, \varepsilon)$. We then consider reforms to $T_e$ by constructing the composite schedule $T_p = T_e + T_d$. While $T_e$ accurately reflects the full heterogeneity in the actual system we will restrict ourselves to reforms where $T_d$ belongs to a particular parametric class. The parameters of $T_d$ are chosen to maximise the revenue of the government $R(T_d)$:

$$R(T_d) = \int_{X,\epsilon} \int_{\varepsilon} [T_e(wh^*, \mathbf{h}^*; X) + T_d(wh^*, \mathbf{h}^*; X)] dF(\varepsilon) dG(X, \epsilon) \tag{2}$$

subject to the requirement that each individual is at least as well off as under the actual tax and transfer system $T_e$:

$$U(T_e + T_d, X, \epsilon, \varepsilon) \geq U(T_e, X, \epsilon, \varepsilon) \; \forall \; (X, \epsilon, \varepsilon), \tag{3}$$

and where $U(T_e + T_d, X, \epsilon, \varepsilon)$ denotes maximised utility under the reformed system. If revenue is not maximised under $T_e$ then it can not be Pareto efficient, since it would be possible to reform the system in a direction which, by raising revenue, allows the welfare of some individuals to be improved without harming others. Note that Pareto improvements in this setting require *reductions* in tax schedules.

## 2.2 Social Welfare Improving Reforms

The second stage of our policy analysis maintains the same positive aspects as described above, but introduces an alternative normative framework. It concerns the choice of a tax schedule $T$ in which the government is allocating a fixed amount of revenue $R$ to a specific demographic group in a way which will maximise the social welfare for this group. Such a schedule balances redistributive objectives with efficiency considerations. Redistributive preferences are represented through the social welfare function $W$, defined as the sum of transformed individual utilities:

$$W(T) = \int_{X,\epsilon} \int_{\varepsilon} \Upsilon(U(c(h^*; T, X, \epsilon), h^*; X, \epsilon, \varepsilon)) dF(\varepsilon) dG(X, \epsilon) \tag{4}$$

where for a given cardinal representation of $U$, the utility transformation function Y determines the governments relative preference for the equality of utilities. This maximization is subject to the incentive compatibility constraint which states that lone mothers choose their hours of work optimally given $T$ (as in equation 1) and the government resource constraint:

$$\int_{X,\epsilon} \int_{\varepsilon} T(wh^*, \mathbf{h}^*; X) dF(\varepsilon) dG(X, \epsilon) \geq \overline{T} (\equiv -R). \tag{5}$$

As in our exploration of Pareto improving reforms, we will restrict $T$ to belong to a particular parametric class of tax functions. This is discussed in section 7 when we empirically examine the optimal design of the UK tax and transfer schedule.

# 3 Tax Credit Reform and Low Income Support

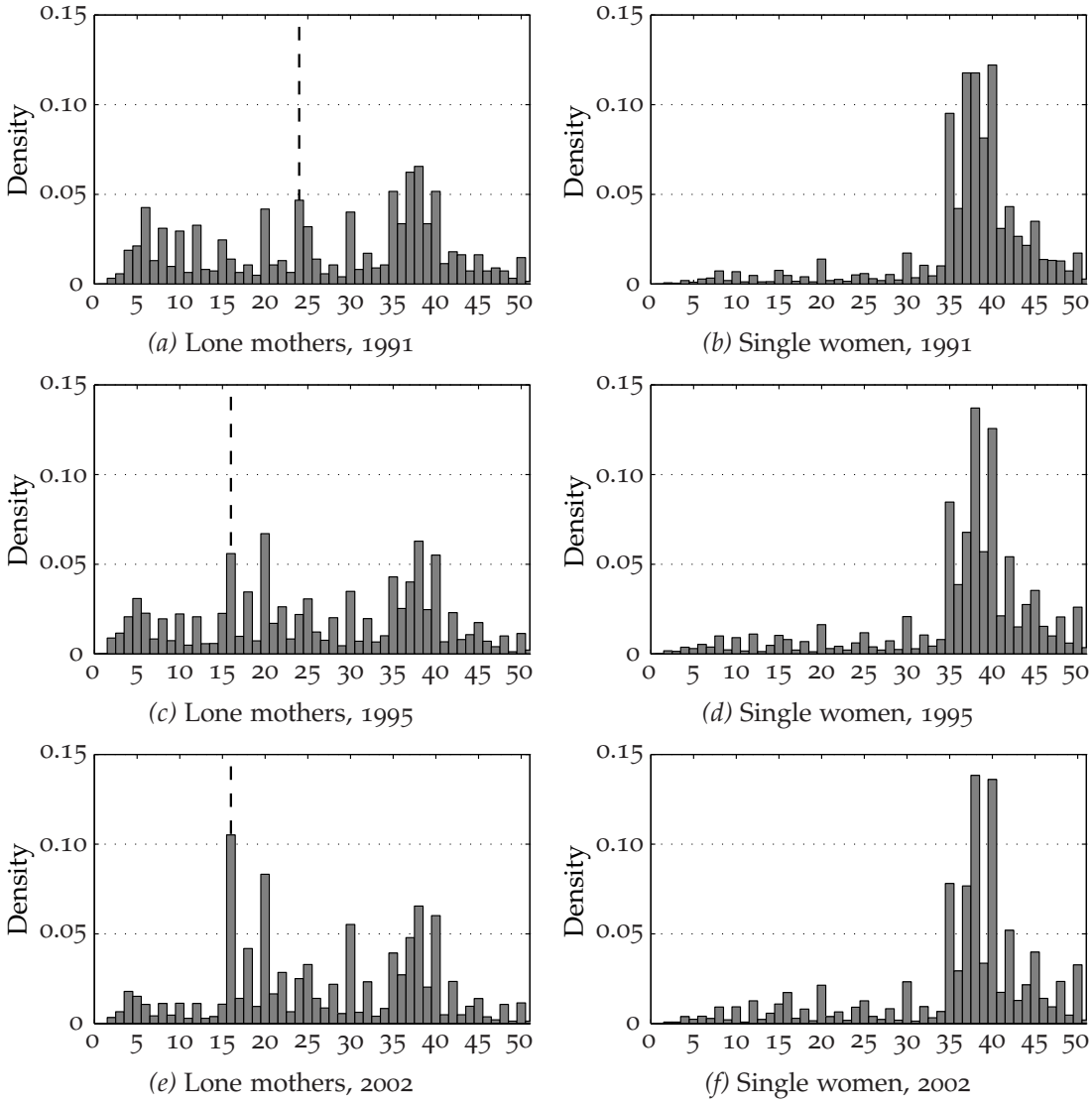
The increasing reliance on tax-credit policies during the 1980s and 1990s, especially in the UK and the US, reflected the secular decline in the relative wages of low skilled workers with low labour market attachment together with the growth in single-parent households (see Blundell, 2002, and references therein). The specific policy context for this paper is the Working Families' Tax Credit (WFTC) reform which took place in the UK at the end of 1999. A novel feature of the British tax credit system is that it makes use of minimum hours conditions in addition to an employment condition. Specifically, WFTC eligibility required a working parent to record at least 16 hours of work per week. Moreover, there was a further hours contingent bonus for working 30 hours or more.

As in the US, the UK has a long history of in-work benefits, starting with the introduction of Family Income Supplement (FIS) in 1971. In 1988 FIS became Family Credit (FC), and in October 1999 this was replaced by Working Families' Tax Credit. While these programmes have maintained a similar structure, the reforms have been associated with notable increases in their generosity. As described above, an important feature of British programmes of in-work support since their inception – and in contrast with programmes such as the US Earned Income Tax Credit – is that awards

depend not only on earned and unearned income and family characteristics, but also on a minimum weekly hours of work requirement. Originally set at 24 hours per week, this was reduced to 16 hours per week in April 1992, where it has stayed since (an additional but smaller credit at 30 hours was introduced in 1995). The impact of this reform to FC on single parents' labour supply is ambiguous: those working more than 16 hours a week had an incentive to reduce their weekly hours to (no less than) 16, while those previously working fewer than 16 hours had an incentive to increase their labour supply to (at least) the new cut-off. Figure 1 shows that the pattern of observed hours of work over this period strongly reflects these incentives. Single women without children were ineligible.

The tax design problem we discuss here draws directly on some of the key features of the WFTC. Indeed, we assess the reliability of our structural labour supply model by its ability to explain behaviour before and after the WFTC reform. The WFTC reform increased the attractiveness of working 16 or more hours a week compared to working fewer hours, and the largest potential beneficiaries of WFTC were those families who were just at the end of the FC benefit withdrawal taper. Conditional on working 16 or more hours, the theoretical impact of WFTC is as follows: (i) people receiving the maximum FC award face an income effect reducing labour supply, but not below 16 hours a week; (ii) people working more than 16 hours and not on maximum FC will face an income effect away from work (but not below 16 hours a week), and a substitution effect towards work; (iii) people working more than 16 hours and earning too much to be entitled to FC but not WFTC will face income and substitution effects away from work if they claim WFTC (see Blundell and Hoynes, 2004). The main parameters of FC and WFTC are presented in the Supplementary Material for this paper (Blundell and Shephard, 2011).

When analysing low income support we take an integrated view of the tax system. This is because tax credit awards in the UK are counted as income when calculating entitlements to other benefits, such as Housing Benefit and Council Tax Benefit. Families in receipt of such benefits would gain less from the WFTC reform than otherwise equivalent families not receiving these benefits; Figure 2 illustrates how the various




*(a)* Lone mothers, 1991 *(b)* Single women, 1991

*(c)* Lone mothers, 1995 *(d)* Single women, 1995

*(e)* Lone mothers, 2002 *(f)* Single women, 2002

*Figure 1:* Female hours of work by survey year. Figure shows the distribution of usual hours of work for women by year and presence of children. Sample is restricted to women aged 18–45. Calculated using UK Labour Force Survey data (for 1991) and UK Quarterly Labour Force Survey data (1995 and 2002). Horizontal axes measure weekly hours of work; the vertical line indicates the minimum hours eligibility.

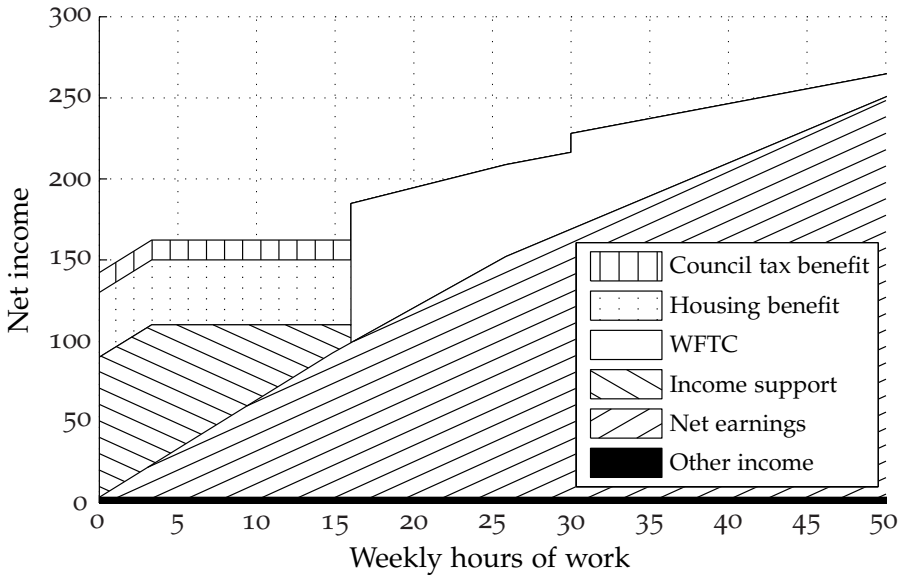



*Figure 2:* Tax and transfer system interactions. Figure shows interaction of tax and transfer system under April 2002 system for a lone parent with a single child aged 5, average band C council tax, £40 per week housing costs, £6 gross hourly wage rate, and no childcare costs. All incomes expressed in April 2002 prices. Calculated using FORTAX.

policies impact on the budget constraint for a low wage lone parent. Moreover, there were other important changes to the tax system affecting families with children that coincided with the expansion of tax credits, and which make the potential labour supply responses considerably more complex. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week).[6]

# 4 A Structural Labour Supply Model

The labour supply specification develops from earlier studies of structural labour supply that use discrete choice techniques and incorporate non-participation in transfer programmes, specifically Hoynes (1996) and Keane and Moffitt (1998). Our aim is to construct a credible model of labour supply behaviour that adequately allows

[6]For many families with children, these increases in out-of-work income meant that replacement rates remained relatively stable.

for individual heterogeneity in preferences and can well describe observed labour market outcomes. As initially discussed in section 2, lone mothers have preferences defined over consumption $c$ and hours of work $h$. Hours of work $h$ are chosen from some finite set $\mathcal{H}$, which in our main empirical results will correspond to the discrete weekly hours points $\mathcal{H} = \{0, 10, 19, 26, 33, 40\}$. These hours points correspond to the empirical hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. In the Supplementary Material we discuss the sensitivity of our results to a finer discretisation of weekly hours; our main results appear robust to this.

## 4.1 Preference Specification

We augment the framework presented in section 2 to allow the take-up of tax-credits to have a direct impact on preferences through the presence of some stigma or hassle cost, and we use $P$ (equal to one if tax credits are received, zero otherwise) to denote the endogenous take-up (programme participation) decision. The utility function is now given by:

$$U(c, h, P; X, \epsilon, \varepsilon) = u(c, h, P; X, \epsilon) + \varepsilon_h,$$

with these preferences allowed to vary with observable demographic characteristics $X$, and vectors of unobservable (to the econometrician) characteristics $\epsilon$ and $\varepsilon$. The state specific errors $\varepsilon$ that are attached to each discrete hours point are assumed to follow a Type-I extreme value distribution.

All the estimation and simulation results presented here assume preferences of the form:

$$u(c, h, P; X, \epsilon) = \alpha_y(X, \epsilon)\frac{c^{\theta_y} - 1}{\theta_y} + \alpha_l(X)\frac{(1 - h/H)^{\theta_l} - 1}{\theta_l} - P\eta(X, \epsilon) \tag{6}$$

where $H = 168$ denotes the total weekly time endowment, and where the set of functions $\alpha_y(X, \epsilon)$, $\alpha_l(X)$ and $\eta(X, \epsilon)$ capture observed and unobserved preference heterogeneity. The function $\eta(X, \epsilon)$ is included to reflect the possible disutility associated with claiming in-work tax credits ($P = 1$), and its presence allows us to rationalise less then complete take-up of tax credits. In each case we allow observed and unobserved heterogeneity to influence the preference shifter functions through

appropriate index restrictions. We parameterise these as:

$$\begin{aligned} \log \alpha_y(X, \epsilon) &= X_y' \beta_y + \epsilon_y \\ \log \alpha_l(X) &= X_l' \beta_l \\ \eta(X, \epsilon) &= X_\eta' \beta_\eta + \epsilon_\eta. \end{aligned}$$

The sets of included demographics are described in section 5.2.

## 4.2 Budget Constraint, Fixed Costs of Work and Childcare Costs

Individuals face a budget constraint, determined by a fixed gross hourly wage rate (generated by a log-linear relationship of the form $\log w = X_w' \beta_w + \epsilon_w$) and the tax and transfer system $T(wh, \mathbf{h}, P; X)$. Non-labour income, such as child maintenance payments, enter the budget constraint through the dependence of the tax and transfer schedule $T$ on demographic characteristics $X$.

We arrive at our measure of consumption $c$ by subtracting both childcare expenditure and fixed work related costs from net income, $wh - T(wh, \mathbf{h}, P; X)$.[7] Both of these processes are described in detail below. Essentially, the choice of work hours will affect consumption through three main channels: firstly, through its direct effect on labour market earnings and its interactions with the tax and transfer system; secondly, through fixed working costs which are payable only if hours of work are strictly positive; thirdly, since working mothers may be required to purchase childcare for their children which varies with maternal hours of employment.

### 4.2.1 Fixed Costs of Work

Fixed work-related costs (as in Cogan, 1981) help provide a potentially important wedge that separates the intensive and extensive margin. They reflect the actual and psychological costs that an individual has to pay to get to work. We model these work-related costs $\alpha_f(h; X)$ as a fixed, one-off, weekly cost subtracted from net

[7] The potential dependence of childcare expenditure on $T$ has been suppressed for simplicity.

income at positive values of working time. We parameterise fixed costs as:

$$\alpha_f(h; X) = \mathbf{1}(h > 0) \times X_f' \beta_f$$

where $\mathbf{1}(\cdot)$ is the indicator function.

### 4.2.2 Childcare Expenditure

Given the rather limited information that our data contains on the types of childcare use, we take a similarly limited approach to modelling, whereby hours of childcare use $h_c$ is essentially viewed as a constraint: working mothers are required to purchase a minimum level of childcare $h_c \geq \alpha_c(h, X, \epsilon)$ which varies stochastically with hours of work and demographic characteristics. Since we observe a mass of working mothers across the hours of work distribution who do not use any childcare, a linear relationship (as in Blundell et al., 2000) is unlikely to be appropriate. Instead, we assume the presence of some underlying latent variable that governs both the selection mechanism and the value of required childcare itself. More specifically, we assume that the total childcare hours constraint is given by:

$$\alpha_c(h, X, \epsilon) = \mathbf{1}(h > 0) \times \mathbf{1}(\epsilon_c > -\beta_c h - \gamma_c) \times (\gamma_c + \beta_c h + \epsilon_c). \tag{7}$$

In our empirical application we will allow all the parameters of this relationship to vary with the set of observable characteristics $X_c$. Total weekly childcare expenditure is then given by $p_c h_c$ with $p_c$ denoting the hourly price of childcare. Empirically, we observe a large amount of dispersion in childcare prices, with this distribution varying systematically with the age composition of children. This is modelled by assuming that $p_c$ follows some distribution $p_c \sim F_c(\cdot; X_c)$ which again varies with demographic characteristics.

### 4.3 Optimal Individual Behaviour

The relationships described above allow us to write consumption at a given hours of work and programme participation combination $(h, P)$ as:

$$c(h, P; T, X, \epsilon) = wh - T(wh, \mathbf{h}, P; X) - p_c \alpha_c(h, X, \epsilon) - \alpha_f(h; X). \tag{8}$$

which may then be substituted into the utility function in equation 6. Each single mother is assumed to jointly choose her hours of work and programme participation decision to maximise her utility. Note that individuals may only be eligible to receive tax credits for some hours choices, and we use $E(h; X, \epsilon)$ to denote such eligibility (equal to one if eligible, zero otherwise). For given hours of work $h$ eligible mothers will elect to receive tax credits if the utility gain from the associated higher consumption level exceeds the utility cost of claiming in-work tax credits. More formally, the optimal programme participation decision $P^*(h)$ will be given by:

$$\begin{aligned} P^*(h) = 1 \quad \text{if } E(h; X, \epsilon) = 1 \quad \text{and} \quad & u(c(h, P = 1; T, X, \epsilon), h, P = 1; X, \epsilon) \\ & \geq u(c(h, P = 0; T, X, \epsilon), h, P = 0; X, \epsilon) \\ P^*(h) = 0 \quad \text{otherwise.} & \end{aligned}$$

It then follows that the optimal (incentive compatible) choice of individual work hours $h^* \in \mathcal{H}$ solves:

$$\max_{h \in \mathcal{H}} U(c(h, P^*(h); T, X, \epsilon), h, P^*(h); X, \epsilon, \varepsilon).$$

## 5 Data and Estimation

### 5.1 Data

We use six repeated cross-sections from the Family Resources Survey (FRS), from the financial year 1997/8 through to 2002/3, which covers the introduction and subsequent expansion of WFTC. The FRS is a cross-section household-based survey drawn from postcode records across Great Britain: around 30,000 families with and without children each year are asked detailed questions about earnings, other forms of

income and receipt of state benefits.

Our sample is restricted to lone mothers who are aged between 18 and 45 at the interview date, not residing in a multiple tax unit household, and not in receipt of any disability related benefits. Dropping families with missing observations of crucial variables, and those observed during the WFTC phase-in period of October 1999 to March 2000 inclusive, restricts our estimation sample to 7,090 lone mothers.

## 5.2 Estimation

The full model (preferences, wages, and childcare) is estimated simultaneously by maximum likelihood; the likelihood function is presented in Appendix A. This simultaneous estimation procedure contrasts with labour supply studies in the UK that have used discrete choice techniques. Perhaps largely owing to the complexity of the UK transfer system, these existing studies (such as Blundell et al., 2000) typically pre-estimate wages which allows net-incomes to be computed prior to the main preference estimation. In addition to the usual efficiency arguments, the simultaneous estimation here imposes internal coherency with regards to the various selection mechanisms. We incorporate highly detailed representations of the tax and transfer system using FORTAX (Shephard, 2009). The budget constraint varies with individual circumstances, and reflects the complex interactions between the many components of the tax and transfer system. To facilitate the estimation procedure, the actual tax and transfer schedules are modified slightly to ensure that there are no discontinuities in net-income as either the gross wage or childcare expenditure vary for given hours of work. We do not attempt to describe the full UK system here, but the interested reader may consult Adam and Browne (2009) and O'Dea et al. (2007) for recent surveys; see Shephard (2009) for a discussion of the implementation of the UK system in FORTAX.

The set of demographics characteristics contained in both $X_y$ and $X_l$, and therefore affecting the marginal rate of substitution between consumption and leisure, are age, education (a zero-one dummy equal to one if the individual had completed compulsory schooling), number of children, and a series of dummies for age-of-youngest

child (0–4, 5–10, and with 11–18 as the omitted category). Age and education also affect the cost of accessing tax credits through $X_\eta$, as does ethnicity and the currently operating tax credit programme. We model this by including a zero-one dummy for the entire WFTC period, together with an additional variable to capture possible first year introductory effects. The wage equation regressors $X_w$ comprise the age that education was completed, a polynomial in age, ethnicity, region, and a series of time dummies. Age, education, number of children, age-of-youngest child, ethnicity, and region, are all contained in $X_f$ and so affect the fixed costs of work.

For the purpose of modelling childcare, we define six groups by the age of youngest child (0–4, 5–10, and 11–18) and by the number of children (1 and 2 or more). The stochastic relationship determining hours of required childcare $\alpha_c(h, X, \epsilon)$ varies within each of these groups, as does the childcare price distribution $F_c(\cdot; X_c)$. Using data from the entire sample period, the childcare price distribution is discretised into either four price points (if the youngest child is aged 0–4 or 5–10) or 2 points (if the youngest child is aged 11-18). In each case, the zero price point is included. The positive price points $p_c$ are fixed prior to estimation and correspond to the mid-points in equally sized groups amongst those using paid childcare (these values are presented in the Supplementary Material). The probability that lone mothers face each of these discrete price points is estimated together with the full model.

We impose concavity on the utility function by restricting the power terms $\theta_l$ and $\theta_y$ to be between 0 and 1. The unobserved wage component $\epsilon_w$ and the random preference heterogeneity terms $(\epsilon_y, \epsilon_\eta, \epsilon_c)$ are assumed to be normally distributed. Given the difficulty in identifying flexible correlation structures from observed outcomes (see Keane, 1992), we allow $\epsilon_y$ to be correlated with $\epsilon_w$, but otherwise assume that the errors are independent. The integrals over $\epsilon$ in the log-likelihood function are approximated using Gaussian quadrature with 11 nodes in each integration dimension. See Appendix A for further details.

## 5.3 Specification and Structural Parameter Estimates

The estimates of the parameters of our structural model are presented in the Supplementary Material. The results show that the age of the youngest child has a significant impact on the estimated fixed costs of work $\alpha_f$; fixed work related costs are higher by around £16 per week if the youngest child is of pre-school age. The presence of young children also has a significant effect on the linear preference terms $\alpha_y$ (negatively) and $\alpha_l$ (positively). Parents with more children are also estimated to have a higher valuation for leisure, as well as higher fixed costs of work.

Lone mothers who are older are estimated to have a lower preference for both consumption and leisure, but higher costs of claiming in-work support. Meanwhile, the main impact of education comes through the preference for leisure $\alpha_l$; mothers who have completed compulsory schooling have a lower preference for leisure. Ethnicity enters the model through both fixed costs of work and programme participation costs $\eta$; we find that programme participation costs are significantly higher for non-white lone mothers. These costs are found to fall significantly following the introduction of WFTC, although the reduction in the first year is small (as captured by the inclusion of a first year zero-one dummy variable). In contrast to many theoretical optimal tax studies which assume that preferences are quasi-linear in consumption, our estimate of $\theta_y$ places significant curvature on consumption. The estimate of $\theta_l$ is equal to the upper bound imposed so that estimated preferences are linear in leisure.

Both the intercept $\gamma_c$ and slope coefficient $\beta_c$ in the childcare equation are typically lower for those with older children. This reflects the fact that mothers with older children use childcare less, and that the total childcare required varies less with maternal hours of work. To rationalise the observed distributions, we also require a larger standard deviation $\sigma_c$ for those with older children. As noted in section 5.2, the price distribution of childcare for each group was discretised in such a way that amongst those mothers using paid childcare, there are equal numbers in each discrete price group. Our estimates attach greater probability on the relatively high childcare prices (and less on zero price) than in our raw data. Individuals who do not work are therefore more likely to face relatively expensive childcare were they to work.

The hourly log-wage equation includes the age at which full-time education was completed (which enters positively), and both age and age squared (potential wages are increasing in age, but at a diminishing rate). Lone mothers who reside in the Greater London area have significantly higher wages, and the inclusion of time dummies track the general increase in real wages over time. There is considerable dispersion in the unobserved component of log-wages.

The within sample fit of the model is presented in Tables 1 and 2. The estimated model matches the observed employment states and the take-up rate over the entire sample period very well (see the first two columns of Table 1). We slightly under predict the number of lone mothers working 19 hours per week, and slightly over predict the number working either 26 or 33 hours per week, but the difference is not quantitatively large. Similarly, we obtain very good fit by age of youngest child. The fit to the employment rate is encouraging, and the difference between predicted and empirical hours frequencies never differs by more than around three percentage points and is typically smaller. Furthermore, despite the relatively simple stochastic specification for childcare, our model performs reasonably well in matching both the use of childcare by maternal employment hours (both overall and by age of youngest child), and conditional hours of childcare. Full results are presented in the Supplementary Material.

The fit of the model over time is presented in Table 2. Fitting the model over time is more challenging given that time enters our specification in a very limited manner - through the wage equation and via the change in the stigma costs of the accessing the tax credit. Despite this we are able to replicate the 9 percentage point increase in employment between 1997/98 and 2002/03 reasonably well with our model, although we do slightly under predict the growth in part-time employment over this period.

To understand what our parameter estimates mean for labour supply behaviour we simulate labour supply elasticities under the actual 2002 tax systems across a range of household types. All elasticities are calculated by simulating a 1% increase in consumption at all positive hours points. The results of this exercise are presented in Table 3. Across our sample of single mothers, we obtain an overall participation

*Table 1:* Predicted and empirical frequencies by age of youngest child

| | All | | 0–4 | | 5–10 | | 11–18 | |
|---|---|---|---|---|---|---|---|---|
| | Predicted | Empirical | Predicted | Empirical | Predicted | Empirical | Predicted | Empirical |
| 0 hours | 0.549 | 0.550 | 0.704 | 0.708 | 0.490 | 0.489 | 0.319 | 0.320 |
| | (0.005) | (0.006) | (0.007) | (0.008) | (0.009) | (0.010) | (0.012) | (0.013) |
| 10 hours | 0.078 | 0.068 | 0.063 | 0.049 | 0.090 | 0.083 | 0.086 | 0.081 |
| | (0.003) | (0.003) | (0.004) | (0.004) | (0.004) | (0.005) | (0.006) | (0.007) |
| 19 hours | 0.105 | 0.134 | 0.089 | 0.108 | 0.117 | 0.156 | 0.117 | 0.147 |
| | (0.002) | (0.004) | (0.003) | (0.006) | (0.003) | (0.007) | (0.004) | (0.010) |
| 26 hours | 0.079 | 0.057 | 0.054 | 0.035 | 0.090 | 0.068 | 0.112 | 0.082 |
| | (0.002) | (0.003) | (0.002) | (0.003) | (0.002) | (0.005) | (0.003) | (0.007) |
| 33 hours | 0.087 | 0.077 | 0.048 | 0.042 | 0.099 | 0.086 | 0.152 | 0.136 |
| | (0.002) | (0.003) | (0.002) | (0.004) | (0.003) | (0.005) | (0.004) | (0.009) |
| 40 hours | 0.103 | 0.115 | 0.044 | 0.058 | 0.114 | 0.120 | 0.214 | 0.234 |
| | (0.003) | (0.004) | (0.003) | (0.004) | (0.005) | (0.006) | (0.010) | (0.012) |
| Take-up rate | 0.769 | 0.764 | 0.840 | 0.788 | 0.768 | 0.781 | 0.702 | 0.715 |
| | (0.010) | (0.009) | (0.010) | (0.017) | (0.011) | (0.013) | (0.016) | (0.018) |

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in Section 5.1. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Empirical take-up rates calculated using reported receipt of FC/WFTC with entitlement simulated using FORTAX. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates (presented in the Supplementary Material). Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

*Table 2:* Predicted and empirical frequencies: 1997–2002

| | 1997 | | 2002 | |
|---|---|---|---|---|
| | Predicted | Empirical | Predicted | Empirical |
| 0 hours | 0.595 | 0.600 | 0.493 | 0.507 |
| | (0.007) | (0.014) | (0.007) | (0.013) |
| 10 hours | 0.079 | 0.080 | 0.079 | 0.062 |
| | (0.003) | (0.008) | (0.003) | (0.006) |
| 19 hours | 0.098 | 0.110 | 0.116 | 0.155 |
| | (0.003) | (0.009) | (0.003) | (0.010) |
| 26 hours | 0.069 | 0.043 | 0.090 | 0.063 |
| | (0.002) | (0.006) | (0.002) | (0.007) |
| 33 hours | 0.072 | 0.063 | 0.104 | 0.093 |
| | (0.002) | (0.007) | (0.002) | (0.008) |
| 40 hours | 0.086 | 0.104 | 0.119 | 0.120 |
| | (0.004) | (0.009) | (0.003) | (0.009) |
| Take-up rate | 0.736 | 0.684 | 0.808 | 0.838 |
| | (0.013) | (0.029) | (0.014) | (0.016) |

Notes: See notes accompanying Table 1.

elasticity of 0.77, with our estimates implying a lower participation elasticity for single mothers whose youngest child is under 4 (an elasticity of 0.66), while they are significantly higher for mothers with school aged children (0.90 if youngest child is aged 5-10; 0.75 if the youngest child is aged 11-18). Intensive elasticities, which here measure the responsiveness of hours worked amongst employed single mothers to changes in in-work consumption, are small and are also increasing for parents with older children. Since mothers with older children also work more hours on average (see Table 1), these intensive elasticities also reflect larger increases in absolute hours for these groups. Compensated intensive elasticities are slightly higher. Finally, the total hours elasticities reported in the table combine these intensive and extensive responses.[8] Here, the lower employment rates for single mothers with younger children produces somewhat higher total hours elasticities for these groups.[9]

[8]The total hours elasticity $\eta_t$ is related to the intensive and extensive elasticities (respectively $\eta_e$ and $\eta_i$) according to $\eta_t = \eta_i + (Q/P) \times \eta_e$. Here, $P$ denotes the employment rate, and $Q$ is the ratio of average hours of new workers, relative to the initial average hours of existing workers.

[9]A large participation (extensive) elasticity and a relatively small intensive elasticity have been reported in other studies, see Blundell and Macurdy (1999). A useful recent reference is Bishop et al. (2009) who report a (fitted) intensive elasticity of 0.05 in 2003, as well as a (fitted) participation elasticity of 0.25 in the same year, for single mothers in the US.

*Table 3:* Simulated elasticities

| | All | | 0–4 | | 5–10 | | 11–18 | |
|---|---|---|---|---|---|---|---|---|
| | Uncomp. | Comp. | Uncomp. | Comp. | Uncomp. | Comp. | Uncomp. | Comp |
| Participation | 0.770 | 0.770 | 0.663 | 0.663 | 0.897 | 0.897 | 0.745 | 0.745 |
| Intensive | 0.042 | 0.123 | 0.032 | 0.094 | 0.043 | 0.128 | 0.047 | 0.136 |
| Total Hours | 1.534 | 1.616 | 2.253 | 2.317 | 1.590 | 1.676 | 1.007 | 1.097 |

Notes: All elasticities simulated under actual 2002 tax systems with complete take-up of WFTC. Elasticities are calculated by increasing consumption by 1% at all positive hours choices. Participation elasticities measure the percentage point increase in the employment rate; intensive elasticities measure the percentage increase in hours of work amongst workers in the base system; total hours elasticities measure the percentage increase in total hours

### 5.4 Simulating the WFTC Reform

Before we proceed to consider optimal design problems using our structural model, we first provide an evaluation of the impact of the WFTC reform described in section 3 on single mothers. This exercise considers the impact of replacing the actual 2002 tax systems with the April 1997 tax system on the 2002 population. This exercise is slightly different to simply examining the change in predicted states over this time period as it removes the influence of changing demographic characteristics.

The full results of this policy reform simulation are presented in the Supplementary Material. Overall we predict that employment increased by 5 percentage points as a result of these reforms, with the increase due to movements into both part-time and full-time employment. Comparing with Table 2 we find the reform explains over a half of the rise in employment over this period. The predicted increase in take-up of tax credits is also substantial, with this increase driven both by the changing entitlement and the estimated reduction in programme participation costs.

## 6 Pareto Improving Reforms

In this section we use our structural model to examine the efficiency of the actual 2002 tax and transfer system $T_e$ for single mothers with one child (and with complete take up of tax credits). We will first restrict ourselves to reforms where the change

in the tax schedule $T_d$ is a function only of earnings $wh$; later we will also allow it to be a function of partially observed hours of work. To identify the regions where Pareto improvements are attainable we specify $T_d$ as a flexible piecewise linear function of earnings. This schedule is characterised by a uniform change in out-of-work income, together with twenty-one different marginal tax rates. These marginal tax rates, which are restricted to lie between -100% and 100%, apply to weekly earnings from £0 to £500 in increments of £25, and then all weekly earnings above £500.

As described in section 2.1 we search for the parameters of this schedule which maximise the revenue of the government, subject to the requirement that each individual is at least as well off as under the actual tax and transfer systems $T_e$. That is, we require that $U(T_e + T_d, X, \epsilon, \varepsilon) \geq U(T, X, \epsilon, \varepsilon)$ for all $(X, \epsilon, \varepsilon)$. Recall that Pareto improvements in this setting require *reductions* in tax schedules.

## 6.1 Efficiency Implications for the Tax Schedule

The results of this exercise are presented in column 2 of Table 4. Reductions in the tax schedule are found for weekly earnings between 225 and 400 pounds per week. This is precisely the range where the density of earnings is falling most quickly (see column 1 of the same table). As Werning (2007) notes, reductions in the tax schedule at a point will cause some individuals to reduce their labour supply, and others to increase it. While tax revenue is always lost from the former group, it can be increased for the latter. If the earnings density is falling sufficiently quickly, then the number of individuals who increase their labour supply will be large relative to the number who decrease it, making an increase in tax revenue more likely.

The table also quantifies the inefficiency under the existing system by comparing the actual and maximised revenue levels from this exercise. The same metric was proposed by Werning (2007) but was not quantitatively explored. As a result of this reform, we find that the government expenditure on single mothers is reduced by around 0.1%. Thus, the increase in tax revenue that this particular reform delivers is clearly very small and suggests that the actual system is close to being efficient. Of course, this metric does not quantify any gains that accrue to single mothers as a

result of the reductions in the tax schedules that they face.

Before we explore incorporating partial hours observability into $T_d$ we first consider a somewhat more relaxed criterion where we integrate over some dimensions of the unobserved heterogeneity and require that individuals are made no worse off for all $(X, \epsilon_w)$. The set of inequality constraints (equation 3) are then replaced by:

$$\int_{\epsilon_{-w}} \int_{\varepsilon} U(T_e + T_d, X, \epsilon, \varepsilon) dF(\varepsilon) dG(X, \epsilon_{-w}|\epsilon_w) \geq \int_{\epsilon_{-w}} \int_{\varepsilon} U(T_e, X, \epsilon, \varepsilon) dF(\varepsilon) dG(X, \epsilon_{-w}|\epsilon_w)$$

for all $(X, \epsilon_w)$. This may be viewed as an appropriate criterion if we think of welfare conditional on characteristics $X$ and idiosyncratic productive capacity $\epsilon_w$. Note that this relaxed criterion does not necessarily require reductions in the tax schedule everywhere. The results are shown in column 4 of Table 4, and are extremely similar to those obtained in our initial exercise.

## 6.2 Incorporating Hours Information

We now consider the use of hours information to improve efficiency. The hours rules in $T_d$ are restricted to operate at the same location as under the actual system $T_e$ (that is, further payments are received if working at the discrete points corresponding to more than 16 and more than 30 hours per-week). Note that if we condition on all the observed and unobserved heterogeneity, then Pareto improvements do not permit any reductions in these hours contingent payments since it would make individuals with a particularly high attachment to a given hours state worse off. This severely limits the potential for reforms to the hours rules to yield Pareto improvements. Indeed, the revenue maximizing tax schedules (column 3) does not alter the hours bonuses, with the reformed schedule the same as reported in column 2 of the same table.

Unsurprisingly, the more relaxed criterion produces quite different results as we are integrating over the unobserved heterogeneity $\varepsilon$ that is responsible for this hours attachment. The results from this exercise (see column 5) point to a small increase in out-of-work income, together with a reduction in the size of the part-time hours bonus and a large increase in the full-time hours bonus. There are also pronounced

*Table 4:* Pareto improving changes to the tax schedule

| Weekly | Base | Conditional on $(X, \epsilon, \varepsilon)$ | | Conditional on $(X, \epsilon_w)$ | |
|---|---|---|---|---|---|
| Earnings | Density | No hours rule | Hours rule | No hours rule | Hours rule |
| 0–25 | 0.005 | 0.000 | 0.000 | 0.000 | -0.297 |
| 25–50 | 0.041 | 0.000 | 0.000 | 0.000 | 0.243 |
| 50–75 | 0.046 | 0.000 | 0.000 | 0.000 | 0.194 |
| 75–100 | 0.044 | 0.000 | 0.000 | 0.000 | -0.119 |
| 100–125 | 0.054 | 0.000 | 0.000 | 0.000 | 0.025 |
| 125–150 | 0.048 | 0.000 | 0.000 | 0.000 | 0.192 |
| 150–175 | 0.049 | 0.000 | 0.000 | 0.000 | -0.231 |
| 175–200 | 0.042 | 0.000 | 0.000 | 0.000 | -0.075 |
| 200–225 | 0.034 | -0.076 | -0.076 | -0.083 | 0.167 |
| 225–250 | 0.032 | 0.077 | 0.077 | 0.088 | -0.048 |
| 250–275 | 0.021 | -0.435 | -0.435 | -0.456 | -0.092 |
| 275–300 | 0.020 | 0.064 | 0.064 | 0.074 | -0.107 |
| 300–325 | 0.016 | -0.073 | -0.073 | -0.052 | 0.072 |
| 325–350 | 0.018 | 0.273 | 0.273 | 0.167 | 0.074 |
| 350–375 | 0.010 | 0.170 | 0.170 | 0.253 | 0.193 |
| 375–400 | 0.008 | 0.000 | 0.000 | 0.059 | 0.224 |
| 400–425 | 0.008 | 0.000 | 0.000 | 0.026 | 0.107 |
| 425–450 | 0.007 | 0.000 | 0.000 | -0.030 | -0.354 |
| 450–475 | 0.007 | 0.000 | 0.000 | -0.038 | 0.178 |
| 475–500 | 0.006 | 0.000 | 0.000 | -0.001 | -0.001 |
| 500+ | 0.027 | 0.000 | 0.000 | 0.000 | -0.269 |
| Out-of-work Income | | 0.000 | 0.000 | 0.000 | 0.269 |
| Bonus at 16 hours | | – | 0.000 | – | -1.370 |
| Bonus at 30 hours | | – | 0.000 | – | 18.616 |
| *Change in expenditure* | | -0.090% | -0.090% | -0.095% | -0.692% |

Notes: Table presents changes to the structure of marginal tax rates, out-of-work income, and hours contingent payments that yield Pareto improvements conditional on $(X, \epsilon, \varepsilon)$ and $(X, \epsilon_w)$ respectively. The base system refers to the actual 2002 tax and transfer system with complete take-up of tax credits. All incomes are in pounds per week and are expressed in April 2002 prices.

changes to marginal tax rates over the entire distribution of labour earnings. This reform produces larger reductions in government expenditure relative to when we did not adjust the size of the hours bonuses (around 1%). The requirement that no individual is made worse off following a tax reform is a demanding criterion, particularly in the presence of preference heterogeneity. In the Supplementary Material we quantify the extent to which imposing this requirement may restrict the potential for the type of social welfare improving reforms that we consider in the following section.

# 7 Optimal Design of the Tax and Transfer Schedule

In this section we consider the normative implications when we adopt a social welfare function with a set of subjective social welfare weights. The analysis here shows the key importance of the differences in labour supply responses at the extensive and intensive margin. We also examine the welfare cost from moving to an administratively simpler linear tax system. The variation in response elasticities noted in our discussion of the estimated model above points to potential gains from allowing the optimal schedule to vary with children's age. We investigate such a design.

Given the use of a minimum hours condition for eligibility in the British tax credit system, we also consider the design in the case of a minimum hours rule. We show that if hours of work are partially (but otherwise accurately) observable, then there can be welfare gains from introducing an hours rule for lone mothers. However, accurately observing hours of work is crucial for this result. Our results suggest that if hours of work are subject to random measurement error or direct misreporting then the welfare gains that can be realised may be much reduced. Our analysis here therefore supports the informal discussion regarding the inclusion of hours in the tax base in Banks and Diamond (2010). Before detailing these results, we first turn to the choice of social welfare transformation and the parameterisation of the tax and transfer schedule.

## 7.1 Optimal Tax Specification

To implement the optimal design analysis described in section 2.2 we approximate the underlying non-parametric optimal schedule by a piecewise linear tax schedule as in section 6. Here the tax schedule will be characterised by a level of out-of-work income (income support), and nine different marginal tax rates.[10] We do not tax any non-labour sources of income, and do not allow childcare usage to interact with tax and transfer schedule unless explicitly stated. When we later allow for partial observability of hours we introduce additional payments that are received only if the

[10]These marginal tax rates are again restricted to lie between -100% and 100%, but now apply to weekly earnings from £0 to £400 in increments of £50, and then all weekly earnings above £400.

individual fulfills the relevant hours criteria.

In all of these illustrations we continue to condition upon the presence of a single child, and set the value of government expenditure equal to the predicted expenditure on this group within our sample. Conditioning upon this expenditure we numerically solve for the tax and transfer schedule that maximises social welfare. Throughout this section we adopt the following utility transformation in the social welfare function:

$$\Upsilon(U;\theta) = \frac{(\exp U)^{\theta} - 1}{\theta}, \tag{9}$$

which controls the preference for equality by the parameter $\theta$ and also permits negative utilities which is important in our analysis given that the state specific errors $\varepsilon$ can span the entire real line. When $\theta$ is negative, the function in equation 9 favours the equality of utilities; when $\theta$ is positive the reverse is true. By L'Hôpital's rule $\theta = 0$ corresponds to the linear case. Note that $-\theta = -\Upsilon''(U;\theta)/\Upsilon'(U;\theta)$ so that $-\theta$ can be interpreted as the coefficient of absolute inequality aversion.

We solve the schedule for a set of parameter values $\theta = \{-0.4, -0.2, 0.0\}$ and then derive the social weights that characterise these redistributive preferences. We do not consider cases where $\theta > 0$. The presence of state specific Type-I extreme value errors, together with our above choice of utility transformation has some particularly convenient properties, as the following Proposition now demonstrates.

**Proposition 1.** *Suppose that the utility transformation function is as specified in equation (9). If $\theta = 0$ then conditional on $X$ and $\epsilon$ the integral over (Type-I extreme value) state specific errors $\varepsilon$ in equation (4) is given by:*

$$\log\left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right) + \gamma$$

*where $\gamma \approx 0.57721$ is the Euler-Mascheroni constant. If $\theta < 0$ then conditional on $X$ and $\epsilon$ the integral over state specific errors is given by:*

$$\frac{1}{\theta}\left[\Gamma(1-\theta) \times \left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right)^{\theta} - 1\right]$$

*where* $\Gamma$ *is the gamma function.*

*Proof.* The result for $\theta = 0$ follows directly from an application of L'Hôpital's rule, and the well known result for expected utility in the presence of Type-I extreme value errors (see McFadden, 1978). See Appendix B for a proof in the case where $\theta < 0$. □

This proposition, which essentially generalizes the result of McFadden (1978), facilitates the numerical analysis as the integral over state specific errors does not require simulating. Moreover, the relationship between the utilities in each state, and the contribution to social welfare for given $(X, \epsilon)$ is made explicit and transparent.

## 7.2 Implications for the Tax Schedule

The underlying properties from the labour supply model, together with the choice of social welfare weights, are the key ingredients in the empirical design problem. As set out in section 2, our model is characterised by both intensive and extensive labour supply responses. The summary labour supply elasticity measures presented in Table 3 point to a sizeable extensive elasticity and a relatively small intensive elasticity. As formalised by Saez (2002), whenever extensive labour supply responses are high at low earnings relative to intensive responses, low (or even negative) marginal tax rates are more likely to be optimal. Starting from an initially high marginal tax rate, the marginal cost associated with a reduction in this tax rate (higher earners reducing their labour supply on the intensive margin), is likely to be dominated by the marginal benefit (by encouraging non-workers to enter work).

The parameter estimates presented in the Supplementary Material (and discussed in section 5.3) show that both observed and unobserved preference heterogeneity are important determinants of individual utilities. Even when only intensive labour supply responses are permitted, the presence of such multi-dimensional heterogeneity (preferences and ability) exerts an important influence on the structure of tax rates, and can provide another source of departure from the predictions of the standard Mirrlees (1971) model. Choné and Laroque (2010) demonstrate that the optimal schedule depends on the average social weights of individuals conditional on observed earnings. The precise influence of heterogeneity then depends on how its

*Table 5:* Social welfare weights under optimal system

| Weekly | $\theta = -0.4$ | | $\theta = -0.2$ | | $\theta = 0.0$ | |
|---|---|---|---|---|---|---|
| Earnings | Density | Weight | Density | Weight | Density | Weight |
| 0 | 0.398 | 1.378 | 0.367 | 1.305 | 0.281 | 1.073 |
| 0–50 | 0.055 | 1.340 | 0.051 | 1.218 | 0.039 | 0.968 |
| 50–100 | 0.109 | 1.088 | 0.104 | 1.071 | 0.088 | 0.935 |
| 100–150 | 0.101 | 0.907 | 0.110 | 0.987 | 0.123 | 1.015 |
| 150–200 | 0.100 | 0.718 | 0.111 | 0.855 | 0.136 | 1.024 |
| 200–250 | 0.078 | 0.563 | 0.087 | 0.721 | 0.115 | 1.021 |
| 250–300 | 0.049 | 0.457 | 0.054 | 0.615 | 0.071 | 0.959 |
| 300–350 | 0.043 | 0.347 | 0.046 | 0.504 | 0.060 | 0.945 |
| 350–400 | 0.021 | 0.307 | 0.023 | 0.454 | 0.029 | 0.880 |
| 400+ | 0.046 | 0.184 | 0.047 | 0.305 | 0.058 | 0.806 |

Notes: Table presents social welfare weights under optimal structure of marginal tax rates and out-of-work income under range of distributional taste parameters $\theta$ as presented in Table 6. All incomes are in pounds per week and are expressed in April 2002 prices. Welfare weights are obtained by increasing consumption uniformly in the respective earnings range and calculating a derivative; weights are normalized so that the earnings-density-weighted sum under optimal system is equal to unity.

distribution varies with earnings; they also show that there are conditions under which negative marginal tax rates may become optimal in this setting.

We now describe our results. For the choice of utility transformation function in equation (9) we examine the impact of alternative $\theta$ values. In Table 5 we present the underlying average social welfare weights evaluated at the optimal schedule (discussed below) according to these alternative $\theta$ values. For all three values of $\theta$ considered here the weights are broadly downward sloping. For the most part we focus our discussion here on the -0.2 value, although we do provide a sensitivity of our results to the choice of $\theta$ and find the broad conclusions are robust to this choice.

In the first three columns of Table 6 we present the optimal tax and transfer schedules across the alternative $\theta$ values; these schedules are also illustrated in Figure 3. In the table we present standard errors for the parameters of the optimal tax schedule, which are obtained by sampling 500 times from the distribution of parameter estimates and re-solving for the optimal schedule conditional on the sample distribution of covariates. In all the simulations performed here, we obtain a broadly progressive marginal tax rate structure: marginal tax rates are typically much lower in the first tax

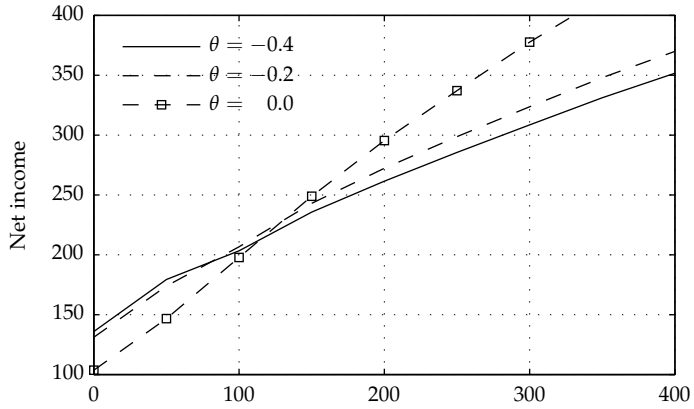



*Figure 3:* Optimal tax schedules with alternative values of $\theta$. All incomes are measured in April 2002 prices and are expressed in pounds per week.

bracket (earnings up to £50 per-week) than at higher earnings. Apart from the $\theta = 0.0$ case, the calculated marginal tax rates are much higher in the second bracket than the first, but then fall before proceeding to generally increase with labour earnings. As we increase the value of $\theta$ (less redistributive concern), we obtain reductions in the value of out-of-work income. This is accompanied by broad decreases in marginal tax rates, except in the first tax bracket where marginal tax rates are largely unchanged. The social welfare weights presented in Table 5 reflect these changes.[11]

The results presented in Table 6 point towards a non-linear tax schedule over a large range of earnings. For each value of $\theta$ considered we quantify the welfare gains from allowing for such non-linearity by calculating the increase in government expenditure required such that the value of social welfare under the optimal linear tax system is the same as under the non-linear systems above. This produces optimal constant marginal tax rates of 43.5%, 37.6% and 11.3% (for $\theta = -0.4$, $\theta = -0.2$ and $\theta = 0.0$ respectively). In the illustrations when $\theta = -0.2$, government expenditure would need to increase by 1.5% to achieve the same level of social welfare.

[11]Comparing actual tax schedules to the optimal schedules from Table 6 is complicated as the actual systems vary in multiple dimensions. Broadly speaking, the optimal tax schedule (when $\theta = -0.2$) has higher (lower) values of out-of-work support than the actual April 2002 system for families with low (high) values of housing rent and Council Tax. For low values of earnings we obtain lower marginal tax rates (except at very low earnings due to an income disregard in Income Support). For lone mothers with moderate wages we obtain lower marginal tax rates over a large range of earnings.

*Table 6:* Optimal tax schedules

| Weekly | No hours | | | 19 hours | | | Optimal hours | | |
|---|---|---|---|---|---|---|---|---|---|
| Earnings | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ |
| 0–50 | 0.132 | 0.144 | 0.139 | 0.266 | 0.280 | 0.252 | 0.053 | 0.056 | 0.072 |
| | (0.028) | (0.025) | (0.029) | (0.029) | (0.034) | (0.037) | (0.031) | (0.028) | (0.028) |
| 50–100 | 0.520 | 0.344 | -0.022 | 0.995 | 0.899 | 0.328 | 0.778 | 0.646 | 0.295 |
| | (0.030) | (0.030) | (0.044) | (0.006) | (0.034) | (0.062) | (0.030) | (0.032) | (0.044) |
| 100–150 | 0.354 | 0.275 | -0.022 | 0.466 | 0.355 | -0.013 | 0.535 | 0.481 | 0.267 |
| | (0.019) | (0.020) | (0.037) | (0.027) | (0.019) | (0.039) | (0.021) | (0.022) | (0.030) |
| 150–200 | 0.483 | 0.414 | 0.069 | 0.503 | 0.440 | 0.090 | 0.698 | 0.650 | 0.321 |
| | (0.014) | (0.017) | (0.033) | (0.014) | (0.017) | (0.035) | (0.028) | (0.030) | (0.050) |
| 200–250 | 0.520 | 0.471 | 0.167 | 0.535 | 0.484 | 0.173 | 0.672 | 0.638 | 0.338 |
| | (0.015) | (0.017) | (0.038) | (0.015) | (0.017) | (0.039) | (0.030) | (0.032) | (0.051) |
| 250–300 | 0.540 | 0.501 | 0.189 | 0.551 | 0.512 | 0.197 | 0.659 | 0.632 | 0.338 |
| | (0.020) | (0.021) | (0.040) | (0.020) | (0.022) | (0.042) | (0.043) | (0.045) | (0.060) |
| 300–350 | 0.546 | 0.514 | 0.266 | 0.554 | 0.521 | 0.270 | 0.644 | 0.618 | 0.365 |
| | (0.023) | (0.025) | (0.053) | (0.024) | (0.026) | (0.053) | (0.038) | (0.040) | (0.064) |
| 350–400 | 0.590 | 0.561 | 0.285 | 0.604 | 0.575 | 0.293 | 0.728 | 0.715 | 0.458 |
| | (0.019) | (0.020) | (0.040) | (0.019) | (0.021) | (0.042) | (0.029) | (0.031) | (0.054) |
| 400+ | 0.616 | 0.599 | 0.401 | 0.623 | 0.607 | 0.403 | 0.687 | 0.676 | 0.477 |
| | (0.008) | (0.009) | (0.023) | (0.008) | (0.009) | (0.024) | (0.008) | (0.009) | (0.029) |
| Out-of-work Income | 135.975 | 131.170 | 103.651 | 136.226 | 131.361 | 104.407 | 137.262 | 132.204 | 106.153 |
| | (s1.672) | (s1.680) | (s3.308) | (1.704) | (1.686) | (3.348) | (1.740) | (1.736) | (3.300) |
| Hours bonus | – | – | – | 36.290 | 38.698 | 23.231 | 44.056 | 48.632 | 47.995 |
| | | | | (1.670) | (1.357) | (2.944) | (2.037) | (1.540) | (5.140) |
| Hours point | – | – | – | 19 | 19 | 19 | 33 | 33 | 33 |

Notes: Table presents optimal structure of marginal tax rates and out-of-work income under range of distributional taste parameters $\theta$. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

## 7.3 Tagging by Age of Child

Before exploring the use of hours contingent payments in the tax schedule we consider how the optimal schedule varies by age of children, should the government decide to condition (or tag) the tax and transfer schedule upon this information. The nature of the optimal income tax schedule in the presence of tagging was explored by Akerlof (1978). Note that WFTC awards depended upon on the age of children (the different rates are presented in the Supplementary Material) as do other parts of the UK tax and transfer system (including Income Support, the main transfer available to low income families working less than 16 hours per week).

Our results in Table 2 suggest that labour supply responses differ significantly at the extensive and the intensive margin according to the age of children. Whenever the labour supply of an identified group is more responsive to tax rates than is that for other groups, then this identified group should face lower marginal tax rates. By shifting the tax burden to otherwise equivalent individuals with lower elasticities of labour supply, the tax structure can create lower efficiency costs while holding unchanged the degree of redistribution from rich to poor, see Gordon and Kopczuk (2010), for example.

In our analysis we do not change the resources going to parents we just adjust the payments according the age of the child. Nonetheless, since our model is static this exercise ignores the dynamics that are introduced by the child ageing process. Clearly, such considerations could be important for the optimal design problem and will be explored in future work. However, this remains an important benchmark case and is likely to still yield important insights, particularly if the population of interest have a sufficiently low discount factor, or are liquidity constrained.

We proceed to solve for the optimal tax schedules for three different groups on the basis of the age of youngest child: under 4, aged 5 to 10 and 11 to 18. Since the childcare requirements of mothers with young children are considerably higher, we also allow for a childcare expenditure subsidy of 70% (which corresponds to the formal childcare subsidy rate under WFTC) to facilitate the comparison of marginal tax rates across these groups. We first solve for these schedules separately when we

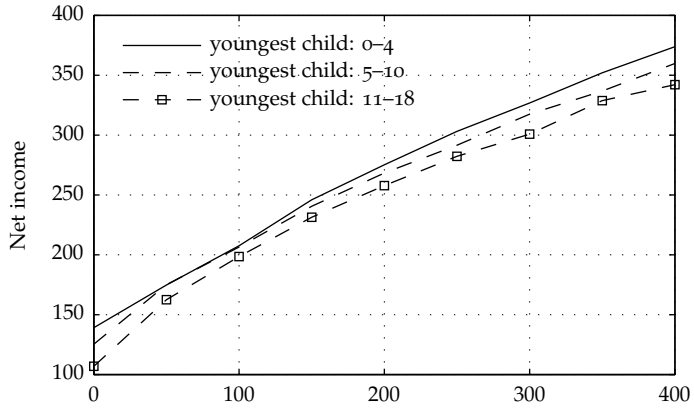



*Figure 4:* Optimal tax schedules by age of child. All schedules are calculated with fixed expenditure division and with $\theta = -0.2$. All incomes are measured in April 2002 prices and are expressed in pounds per week.

condition on the predicted expenditure on each of these groups in our sample; we then solve for these schedules jointly allowing the division of overall expenditure to be re-optimised. Full results are presented in Tables 7a and 7b; Figure 4 illustrates these with fixed group expenditure when $\theta = -0.2$.

While the overall structure of the schedules retain many of the features present in our earlier simulations, our optimal tax simulations here reveal some important differences by the age of children. In the case of fixed within group expenditure (see Table 7a), marginal tax rates tend to be higher at low earnings for lone mothers with younger children: in the first tax bracket marginal tax rates for the youngest group are around 40 percentage points higher than for the oldest group. Amongst women with children from the oldest group we also obtain negative marginal tax rates. The higher marginal tax rates at low earnings for parents with younger children are also accompanied by higher levels of out-of-work support for these groups.

Conditioning upon within group expenditure levels makes an implicit assumption on the weight that the government attaches on the welfare of parents with children of different ages. Under the assumption that the government places equal valuation on these groups we solve for the three schedules jointly (see Table 7b). Relative to the previous simulations, this makes the differences across groups more pronounced. In particular, there are notable increases in expenditure (and out-of-work income levels)

*Table 7:* Optimal tax system by age of child with childcare subsidy (conditional on group expenditure)

*(a)* Fixed expenditure division

| Weekly | 0–4 | | | 5–10 | | | 11–18 | | |
|---|---|---|---|---|---|---|---|---|---|
| Earnings | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ |
| 0–50 | 0.198 | 0.287 | 0.432 | -0.003 | 0.006 | 0.085 | -0.107 | -0.111 | -0.009 |
| 50–100 | 0.503 | 0.344 | 0.043 | 0.545 | 0.370 | 0.013 | 0.478 | 0.279 | -0.013 |
| 100–150 | 0.309 | 0.232 | -0.033 | 0.395 | 0.320 | 0.038 | 0.445 | 0.343 | -0.004 |
| 150–200 | 0.478 | 0.415 | 0.151 | 0.517 | 0.444 | 0.085 | 0.552 | 0.472 | 0.086 |
| 200–250 | 0.490 | 0.442 | 0.149 | 0.579 | 0.537 | 0.265 | 0.577 | 0.510 | 0.154 |
| 250–300 | 0.557 | 0.526 | 0.348 | 0.532 | 0.480 | 0.101 | 0.674 | 0.629 | 0.222 |
| 300–350 | 0.530 | 0.496 | 0.220 | 0.640 | 0.614 | 0.449 | 0.488 | 0.441 | 0.160 |
| 350–400 | 0.592 | 0.563 | 0.384 | 0.583 | 0.540 | 0.168 | 0.771 | 0.734 | 0.383 |
| 400+ | 0.607 | 0.590 | 0.431 | 0.640 | 0.622 | 0.420 | 0.654 | 0.631 | 0.377 |
| Out-of-work income | 140.950 | 139.152 | 126.405 | 131.855 | 125.374 | 95.572 | 118.382 | 106.947 | 66.850 |

*(b)* Optimal expenditure division

| Weekly | 0–4 | | | 5–10 | | | 11–18 | | |
|---|---|---|---|---|---|---|---|---|---|
| Earnings | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ | $\theta = -0.4$ | $\theta = -0.2$ | $\theta = 0.0$ |
| 0–50 | 0.167 | 0.265 | 0.429 | -0.002 | 0.008 | 0.085 | -0.121 | -0.115 | -0.009 |
| 50–100 | 0.535 | 0.368 | 0.047 | 0.536 | 0.362 | 0.016 | 0.441 | 0.254 | -0.024 |
| 100–150 | 0.316 | 0.238 | -0.028 | 0.398 | 0.323 | 0.041 | 0.458 | 0.353 | -0.015 |
| 150–200 | 0.473 | 0.406 | 0.156 | 0.519 | 0.447 | 0.088 | 0.564 | 0.483 | 0.073 |
| 200–250 | 0.482 | 0.433 | 0.153 | 0.584 | 0.541 | 0.268 | 0.585 | 0.517 | 0.146 |
| 250–300 | 0.544 | 0.513 | 0.351 | 0.533 | 0.482 | 0.104 | 0.685 | 0.640 | 0.209 |
| 300–350 | 0.523 | 0.490 | 0.223 | 0.643 | 0.618 | 0.450 | 0.495 | 0.447 | 0.154 |
| 350–400 | 0.581 | 0.551 | 0.387 | 0.585 | 0.543 | 0.171 | 0.780 | 0.742 | 0.372 |
| 400+ | 0.602 | 0.584 | 0.433 | 0.642 | 0.623 | 0.422 | 0.660 | 0.636 | 0.370 |
| Out-of-work income | 156.618 | 154.340 | 123.959 | 127.071 | 120.336 | 93.975 | 100.615 | 90.768 | 71.954 |

Notes: Table presents optimal structure of marginal tax rates and out-of-work income by age of youngest child under range of distributional taste parameters $\theta$. All schedules calculated with an uncapped childcare subsidy equal to 70%. All incomes are in pounds per week and are expressed in April 2002 prices.

for lone mothers with younger children. And while there are some changes in the structure of marginal tax rates (due to income effects) these changes are somewhat smaller in magnitude.

The welfare gains from tagging on the basis of age of children can be calculated in much the same way as when comparing a non-linear schedule to one which is linear. The potential welfare gains appear reasonably large: relative to a system where tagging by the age of youngest child is not possible, government expenditure would have to increase by 2.6% (when $\theta = -0.2$) to obtain the same level of social welfare as that achieved when such tagging is possible. These gains are even larger when more redistributive preferences are considered.

## 7.4 Introducing an Hours Rule

For several decades the UK's tax credits and welfare benefits have made use of rules related to weekly hours of work. As discussed in section 3, individuals must work at least 16 hours a week to be eligible for in-work tax credits, and receive a further credit when working 30 or more hours. While many theoretical models rule out the observability of any hours information, this design feature motivates us to explore the optimal structure of the tax and transfer system when hours can be partially observed as set out in section 2. Essentially, observing some hours of work information allows the government to better distinguish between different types of individual. In the absence of any labour supply participation response, and when the only source of worker heterogeneity is the exogenous wage rate (productive ability), the government is able to redistribute without cost when both hours and earnings are perfectly observable since it can now infer ability. The first best ceases to be attainable once hours of work are only partially observed, but even this information allows the government to better separate types relative to when labour earnings is the only signal.

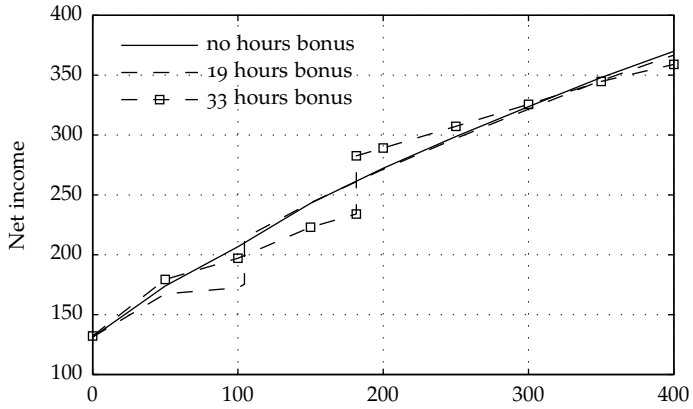
We begin by assuming that the tax authority is able to observe whether individuals are working 19 hours or more, which roughly corresponds to the placement of the main 16 hours condition in the British tax-credit system, and for now we do not allow for any form of measurement error. In this case the tax authority is able to

condition an additional payment on individuals working such hours. The results of this exercise are presented in columns 4–6 in Table 6, and the $\theta = -0.2$ case is also presented in Figure 5a. Relative to the optimal system when such a rule is not implementable, the hours bonus increases marginal rates in the part of the earnings distribution where this hours rule would roughly come into effect (particularly in the £50 to £100 earnings bracket) while marginal rates further up the distribution, as well as the level of out-of-work support, are essentially unchanged. As a result, some non-workers with low potential wages may be induced to work part-time, while some low hours individuals will either not work or increase their hours. Similarly, some high earnings individuals reduce their hours to that required for the bonus.[12]

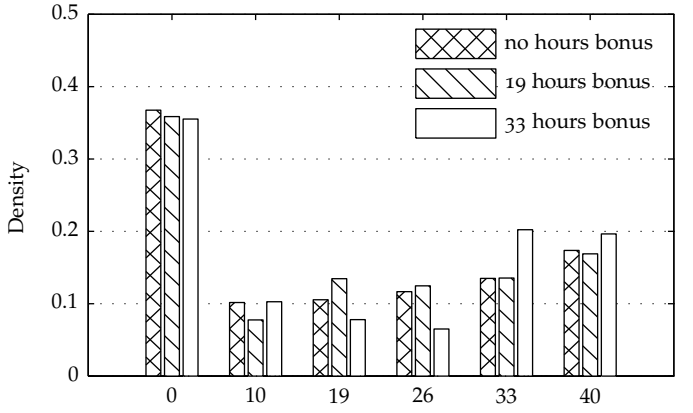
Although there are some notable changes in the structure of the constraint when hours information is partially observable, it does not follow that it necessarily leads to a large improvement in social welfare. Indeed, in the absence of the hours conditioning, there are only few individuals working less than 19 hours (see Figure 5b when $\theta = -0.2$) so the potential that it offers to improve social welfare appear limited. We now provide some guidance concerning the size of the welfare gain from introducing hours rules. The exact experiment we perform is as follows: we calculate the level of social welfare under the optimal schedule with hours contingent payments, and then determine the increase in expenditure that is required to obtain the same level of social welfare in the absence of such hours conditioning; we allow all the parameters of the (earnings) tax schedule to vary so this is obtained at least cost.

Perhaps unsurprisingly, these welfare gains are found to be relatively small. In both the $\theta = -0.4$ and $\theta = -0.2$ cases the expenditure increase required to achieve the level of social welfare obtained under the 19 hours rule is less than 1%. When the least redistributive preferences are considered, this falls to just 0.2%. Even without allowing for any measurement error, it follows that unless the costs of partial hours observability is sufficiently low, it would appear difficult to advocate the use of a 19 hour rule based upon this analysis. This has very important policy implications given that the UK tax credit system makes heavy use of very similar hours conditions. We

[12]The hours bonus is sufficiently large that a mother earning the minimum wage would face an effectively zero participation tax rate at 19 hours.




*(a)* Optimal tax schedules




*(b)* Distribution of work hours

*Figure 5:* Optimal tax schedules with hours bonuses and associated hours distribution. All schedules are calculated with $\theta = -0.2$ and assuming a gross hourly wage of £5.50. All incomes are measured in April 2002 prices and are expressed in pounds per week.

note that Keane and Moffitt (1998) considered introducing a work subsidy in a model with three employment states (non-workers, part-time and full-time work) and multiple benefit take-up. Even small subsidies were found to increase labour supply and to reduce dependence on welfare benefits, and at reduced cost. In contrast to our application (where we are moving from a base with marginal rates well below 100% at low earnings), their simulations considered introducing the subsidy in an environment where many workers faced marginal effective tax rates which often exceeded 100%, and where the receipt of these work subsidies encourages women to exit welfare benefits (and so no longer be affected by the associated stigma costs).

### 7.4.1 An Optimal Hours Rule?

The social welfare gains from introducing a 19 hours rule appear to be only very modest in size at best. In this section we explore whether there are potentially larger gains by allowing the choice of the point at which the hours rule becomes effective to be part of the optimal design problem. The parameters of the optimal tax schedules for all $\theta$ are presented in columns 7–9 of Table 6, while the optimal schedule when $\theta = -0.2$ is also shown in Figure 5a. Apart from when considering the least redistributive government preferences, we obtain an optimal hours rule at the fifth (out of six) discrete hours point, which corresponds to 33 hours per week (when $\theta = 0.0$ the optimal placement shifts to 40 hours per-week). We also note that the size of the optimally placed hours bonus always exceeds that calculated when the hours rule became effective at 19 hours per week.

Introducing an hours rule further up the hours distribution allows the government to become more effective in distinguishing between high wage/low effort and high effort/low wage individuals than at 19 hours to the extent that few higher wage individuals would choose to work very few hours. Relative to the schedule when the hours rule is set at around 19 hours, this alternative placement tends to make people with low and high earnings better off, while people in the middle range lose. While we again obtain very small adjustments to the level of out-of-work income, there are much more pronounced changes to the overall structure of marginal rates. In particular, there are large reductions in the marginal tax rate in the first tax bracket, while marginal rates increase at higher earnings. Figure 5b shows the resulting impact on the hours distribution when $\theta = -0.2$.

As before, we attempt to quantify the benefits from allowing for hours conditioning. Performing the same experiment as we conducted under the 19 hours rule we find that the required increase in expenditure is considerably larger than that obtained previously. We find that a 2.5% increase in expenditure would be required to achieve the same level of social welfare when $\theta = -0.2$ (with very similar increases for the alternative $\theta$ values), which represents a non-trivial welfare gain. In any case, if the government wishes to maintain the use of hours conditional eligibility, the anal-

ysis here suggests that it may be able to improve design by shifting towards a system that primarily rewards full-time rather than part-time work.[13]

### 7.4.2 Measurement Error and Hours Misreporting

The results presented have not allowed for any form of measurement error. While earnings may not always be perfectly measured, it seems likely that there is more scope for mismeasurement of hours as they are conceivably harder to monitor and verify. Indeed, the presence of hours rules in the tax and transfer system presents individuals with an incentive to not truthfully declare whether they satisfy the relevant hours criteria. Relative to when hours are always accurately reported, this would seem to weaken the case for introducing a measure of hours in the tax base as the signal is now less informative about individual type. While we do not explore this issue, we note that the government may be able to improve design by using additional tax instruments that are related to hours of work. An example of such an instrument is childcare expenditure, which may be observed more accurately than self reported hours of work if the tax authorities require expenditure receipts. We quantify the importance of such measurement error by considering two alternative scenarios: firstly, when hours are imperfectly observed due to random measurement error; secondly, when we allow individuals to directly misreport their hours of work to the tax authorities.

In the Supplementary Material we present results from the first case with random measurement error. We show how both the size of the optimal hours bonus and the associated welfare gains decline as reported hours become less informative. Here we focus upon the arguably more plausible case of systematic hours misreporting. We modify our setup by distinguishing between actual hours of work $h$, and reported hours of work $h_R$; actual hours determine both leisure and earnings, while reported hours of work directly affect consumption through the tax schedule, with $T = T(wh, \mathbf{h_R}; X)$. If individuals misreport their hours then they must incur a utility

[13]The welfare gains from a part-time hours rule are also small if we condition by the age of children as described in section 7.3. And while the welfare gains from an optimally placed (full-time) hours rule are also small for mothers with pre-school aged children, these gains are found to be much more substantial for parents with school age children. Full results are available upon request.

cost, which is assumed to be proportional to the distance $h_R - h$. We therefore modify the individual utility function by including $h_R - h$ as an explicit argument, with $U = u(c, h, h_R - h; X, \epsilon) + \varepsilon_h$. This modified utility function is as in equation 6 but now with the additional cost term $b \times (h_R - h)$ subtracted from $u$ whenever $h_R > h$.[14] Misreporting is only possible when $h > 0$, and we refer to the parameter $b \geq 0$ as the misreporting cost. We do not allow individuals to manipulate their earnings $wh$.

As before, we consider tax schedules with a single hours eligibility threshold, and denote this hours requirement as $h_B$. Since misreporting hours is costly, it is only necessary to consider the cases when hours are truthfully revealed $h_R = h$, or when $h_R = h_B > h$. At a given actual hours of work $h < h_B$ individuals will report their hours as $h_R = h_B$ if and only if the utility gain exceeds the cost. That is:

$$u(c(h, T(wh, \mathbf{h_B}; X), X, \epsilon), h, h_B - h; X, \epsilon) > u(c(h, T(wh, \mathbf{h}; X), X, \epsilon), h, 0; X, \epsilon).$$

We present results from this exercise in Table 8.[15] The table illustrates that as the utility cost of misreporting becomes very low, the welfare gain from using reported hours of work diminishes (but the optimal placement remains unchanged for all values considered). Note also that when $b = \infty$ misreporting is never optimal. This analysis suggests that the welfare gains from using hours of work information may be small unless the scope for misreporting hours of work is limited.

# 8 Summary and Conclusions

The aim of this paper has been to examine the optimal design of low income support using a stochastic structural labour supply model. The application focussed on the design of the tax schedule for parents with children, in particular single mothers. The structural labour supply model was shown to be reliable and found to match

[14] In practice misreporting costs are likely to vary with both observed and unobserved worker characteristics. While it is sufficient to model this as a single cost for the purpose of our discussion and simulations here, our framework can easily be extended to incorporate such heterogeneity.

[15] The misreporting cost $b$ is measured relative to standard deviation of the state specific error $\varepsilon$. With an hours bonus payable at 33 hours per week (for example), a value of $b = 0.16$ would mean that the utility cost of reporting 33 hours when actual hours are 26 is equivalent to a $0.16 \times (33 - 26) = 1.12$ standard deviation change in the realisation of the state specific error.

*Table 8:* The effect of hours misreporting on the optimal hours bonus

| Misreporting | $\theta = -0.4$ | | | $\theta = -0.2$ | | | $\theta = 0.0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Cost | bonus | hours | welfare | bonus | hours | welfare | bonus | hours | welfare |
| $\infty$ | 46.53 | 33 | 2.31% | 51.46 | 33 | 2.52% | 54.80 | 40 | 2.57% |
| 0.64 | 46.52 | 33 | 2.31% | 51.45 | 33 | 2.52% | 54.79 | 40 | 2.57% |
| 0.32 | 45.25 | 33 | 2.28% | 49.89 | 33 | 2.50% | 53.76 | 40 | 2.56% |
| 0.16 | 33.73 | 33 | 1.95% | 37.74 | 33 | 2.12% | 41.71 | 40 | 2.16% |
| 0.08 | 24.24 | 33 | 1.36% | 26.54 | 33 | 1.52% | 29.26 | 40 | 1.63% |
| 0.04 | 14.46 | 33 | 0.89% | 15.89 | 33 | 1.00% | 17.41 | 40 | 1.13% |
| 0.02 | 9.24 | 33 | 0.58% | 10.72 | 33 | 0.67% | 12.44 | 40 | 0.83% |
| 0.01 | 7.21 | 33 | 0.43% | 8.12 | 33 | 0.52% | 9.17 | 40 | 0.72% |

Notes: Table shows how the optimal placement and size of hours contingent payments varies with the utility cost of hours misreporting. "Misreporting Cost" refers to the additive utility cost associated with misreporting, and is measured per-hour overstated and relative to standard deviation of the state specific error $\varepsilon$. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

closely the changes in observed behaviour that followed a large reform to the tax credit system in the UK.

The paper has made three contributions to the existing literature on tax design. First, we have taken the structural model of employment and hours of work seriously in designing the optimal schedule of taxes and transfers. To operationalise this we have developed the design problem within an extended Mirrlees framework which has incorporated unobserved heterogeneity, the non-convexities of the tax and transfer system, as well as allowing for childcare costs and fixed costs of work. We first used this model to identify inefficiencies in the actual tax and transfer system and characterised Pareto improving reforms. While this analysis pointed to relatively minor improvements in the UK tax and transfer schedule for lone parents, by imposing a specific social welfare function with reasonable social welfare weights we obtained a reformed non-linear tax schedule with lower tax rates over a large range of earnings for many families, and with tax credits only optimal for low earners.

Tagging has been suggested to improve the trade-off between equality and efficiency. Our second contribution has been to empirically assess the role of tagging taxes by the age of children under a social welfare function. These results highlighted an importance of conditioning effective tax rates on the age of children, with

tax credits being found to be most important for low earning families with school age children. The welfare gains from this age based tagging were also found to be quantitatively significant.

We have noted that hours contingent payments are a key feature in the British tax credit system. Our third contribution was to consider the case where hours of work are partially observable to the tax authorities and to quantify the value of this signal. If the tax authorities are able to choose the lower limit on working hours that trigger eligibility for such families, then we find an empirical case for using a full-time work rule rather than the main part-time rule currently in place for parents in the UK. While this is found to be a more effective instrument, we demonstrate how these welfare gains diminish with both misreporting and measurement error.

# Appendix

# A Likelihood function

In what follows let $\mathcal{P}_j(X, p_{c_k}, \epsilon) \equiv \Pr(h = h_j | X, p_{c_k}, \epsilon)$ denote the probability of choosing hours $h_j \in \mathcal{H}$ conditional on demographics $X$, the childcare price $p_{c_k}$, and the vector of unobserved preference heterogeneity $\epsilon = (\epsilon_w, \epsilon_c, \epsilon_y, \epsilon_\eta)$. Given the presence of state specific Type-I extreme value errors, this choice probability takes the familiar conditional logit form. We also use $\pi_k(X) \equiv \Pr(p_c = p_{c_k} | X)$ to denote the probability of a lone mother with observable characteristics $X$ facing childcare price $p_{c_k}$. In the case of non-workers ($h = h_0$), neither wages nor childcare are observed so that the likelihood contribution is simply given by:

$$\sum_k \pi_k(X) \int_\epsilon \mathcal{P}_0(X, p_{c_k}, \epsilon) dG(\epsilon).$$

Now consider the case for workers when both wages and childcare is observed so that $h_c$ is not censored at zero. Using $E_h \equiv E(h; X, p_c, \epsilon)$ to denote eligibility for in-work support we define the indicator $\mathcal{D}(e, p) = \mathbf{1}(E_h = e, P = p)$. We also let $\Delta u(h_j | p_{c_k}, X, \epsilon_{|\epsilon_\eta = 0})$ denote the (possibly negative) utility gain from claiming in-work

support at hours $h_j$, conditional on demographics $X$, the childcare price $p_{c_k}$, and the vector of unobserved preference heterogeneity $\epsilon$ with $\epsilon_\eta = 0$. Suppressing the explicit conditioning for notational simplicity, the likelihood contribution is given by:

$$\prod_k \pi_k(X)^{\mathbf{1}(p_c = p_{c_k})} \int_{\epsilon_y} \left\{ \mathcal{D}(1,1) \int_{\epsilon_\eta < \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \right.$$
$$\left. + \mathcal{D}(1,0) \int_{\epsilon_\eta > \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \right\}$$
$$dG(\epsilon | \epsilon_w = \log w - X_w' \beta_w, \epsilon_c = h_c - \gamma_c - \beta_c h)$$
$$g_w(\log w - X_w' \beta_w) g_c(h_c - \gamma_c - \beta_c h).$$

If working mothers are not observed using childcare, then $h_c$ is censored at zero and the childcare price also unobserved. Defining $\overline{\epsilon}_c = -\gamma_c - \beta_c h$, the likelihood contribution is given then by:

$$\sum_k \pi_k(X) \iint_{\epsilon_c < \overline{\epsilon}_c, \epsilon_y} \left\{ \mathcal{D}(1,1) \int_{\epsilon_\eta < \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \right.$$
$$\left. + \mathcal{D}(1,0) \int_{\epsilon_\eta > \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \right\}$$
$$dG(\epsilon | \epsilon_w = \log w - X_w' \beta_w) g_w(\log w - X_w' \beta_w).$$

Our estimation also allows for workers with missing wages. This takes a similar form to the above, except that it is now necessary to also integrate over the unobserved component of wages $\epsilon_w$.

All the integration over $\epsilon$ is performed using Gaussian Hermite quadrature with 11 nodes in each integration dimension. When it is unnecessary to integrate over the entire real line in a given dimension, a change of variable is conducted so that integration is performed over $[0, +\infty)$, with appropriate semi-Hermite quadrature formulae then applied.

# B Proof of Proposition

For notational simplicity we abstract from the explicit conditioning of utility on observed and unobserved preference heterogeneity and let $u(h) \equiv u(c(h), h; X, \epsilon)$. We then define $V$ as the integral of transformed utility over state specific errors conditional on $(X, \epsilon)$:

$$V \equiv \int_{\varepsilon} \Upsilon \left( \max_{h \in \mathcal{H}} \left[ u(h) + \varepsilon_h \right] \right) dF(\varepsilon) \tag{A-1}$$

To prove this result we first differentiate $V$ with respect to $u(h)$:

$$\begin{aligned} \frac{\partial V}{\partial u(h)} &= \int_{\varepsilon} \left( \frac{\partial \Upsilon \left( \max_{h \in \mathcal{H}} \left[ u(h) + \varepsilon_h \right] \right)}{\partial u(h)} \right) dF(\varepsilon) \\ &= \int_{\varepsilon} \Upsilon' \left( u(h) + \varepsilon_h \right) \times \mathbf{1} \left( h = \arg\max_{h' \in \mathcal{H}} \left[ u(h') + \varepsilon_{h'} \right] \right) dF(\varepsilon) \end{aligned}$$

Given our choice of utility transformation function in equation 9 and our distributional assumptions concerning $\varepsilon$ the above becomes:

$$\begin{aligned} \frac{\partial V}{\partial u(h)} &= \int_{\varepsilon_h = -\infty}^{\infty} \left\{ e^{(u(h) + \varepsilon_h)} \right\}^{\theta} \left( \prod_{h' \neq h} e^{-e^{-\{\varepsilon_h + u(h) - u(h')\}}} \right) \times e^{-\varepsilon_h} e^{-e^{-\varepsilon_h}} d\varepsilon_h \\ &= \left\{ e^{u(h)} \right\}^{\theta} \int_{\varepsilon_h = -\infty}^{\infty} \left\{ e^{\varepsilon_h} \right\}^{\theta} \times \exp \left( -e^{-\varepsilon_h} \sum_{h' \in \mathcal{H}} e^{-(u(h) - u(h'))} \right) e^{-\varepsilon_h} d\varepsilon_h \end{aligned}$$

We proceed by using the change of variable $t = \exp(-\varepsilon_h)$ so that the above partial derivative becomes:

$$\frac{\partial V}{\partial u(h)} = \left\{ e^{u(h)} \right\}^{\theta} \int_{t=0}^{\infty} t^{-\theta} \times \exp \left( -t \sum_{h' \in \mathcal{H}} e^{-(u(h) - u(h'))} \right) dt$$

By defining $z \equiv t \times \sum_{h' \in \mathcal{H}} e^{-(u(h) - u(h'))}$ we can once again perform a simple change of variable and express the above as:

$$\begin{aligned} \frac{\partial V}{\partial u(h)} &= \left\{ e^{u(h)} \right\}^{\theta} \left\{ \sum_{h' \in \mathcal{H}} e^{-(u(h) - u(h'))} \right\}^{\theta - 1} \int_{z=0}^{\infty} z^{-\theta} e^{-z} dz \\ &= e^{u(h)} \left\{ \sum_{h' \in \mathcal{H}} e^{u(h')} \right\}^{\theta - 1} \int_{z=0}^{\infty} z^{-\theta} e^{-z} dz \\ &= e^{u(h)} \left\{ \sum_{h' \in \mathcal{H}} e^{u(h')} \right\}^{\theta - 1} \Gamma(1 - \theta) \end{aligned} \tag{A-2}$$

where the third equality follows directly from the definition of the Gamma function $\Gamma(\cdot)$. Note that this integral will always converge given that we are considering cases where $\theta < 0$. Integrating equation A-2 we obtain:

$$V = \frac{1}{\theta}\left[\Gamma(1-\theta) \times \left(\sum_{h' \in \mathcal{H}} \exp\left\{u(h')\right\}\right)^{\theta} - 1\right] \tag{A-3}$$

where the constant of integration is easily obtained by considering the case of a degenerate choice set and directly integrating A-1.

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